

Analysis of Interference for a Multi-Radio Channel Assignment Algorithm

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Abstract

Interference is an important parameter that affects the performance of a network. By reducing interference, the capacity of the network is improved and this leads to an increased efficiency in communication. In this paper we analyze the interference of a robust channel assignment algorithm for multi-radio networks. This study is an extension to a previous work in which a channel assignment algorithm that is robust to the presence of primary users was introduced. The previous study was mainly focused on balanced assignment for the robust algorithm, i.e. assigning channels to radios such that all channels are equally distributed in the network. In this paper we give recommendations regarding the most efficient parameters to choose for the algorithm based on the analysis of interference.

keywords: channel assignment, interference analysis, multi-radio network, primary user, energy efficiency

1 Introduction

This paper analyzes the interference for a channel assignment presented in [4], which could be used in a multi-radio multi-channel environment and can provide robustness to primary users that could reclaim multiple channels at once.

The centralized and distributed algorithms presented in [7] require multiple negotiations between nodes and may require cascaded switching of multiple users. These solutions outperform other interference-aware approaches when primary users appear, and achieve similar performance at other times.

Compared to previous channel assignment methods for multi-hop multi-radio networks from [5] and [6], in [7] the channels are carefully assigned so that the primary user's appearance will not partition the network. The algorithms are compared with INSTC [6].

In [1] we introduced a distributed algorithm for channel assignment in a wireless sensor network, in which the monitored area is divided into grids. This grid-based algorithm is robust to the presence of primary users that reclaim one channel at once. It requires the nodes to have GPS or other localization protocols. The distributed

algorithm we introduced in [2] does not require nodes to know their location and it also provides robustness when one channel is reclaimed by primary users.

The algorithm that we analyze in this paper, introduced in [4], is much simpler than the one presented in [7] and it could be used in different kinds of networks, for example mesh networks or networks with limited resources like wireless sensor networks. This algorithm performs simple calculations and a node could assign its channels based on its position in the network. Also, it is robust to the presence of primary users that reclaim multiple channels at once.

Some papers use primary user behavior models but predictability is not always possible. Like [7], we do not assume a predictable primary user activity. But knowing the maximum number of channels that could be reclaimed simultaneously by primary users could help in choosing the best parameters for our algorithm.

The algorithm in [4] could be used for any number of radios and/or channels. We briefly describe this algorithm and then we analyze its interference. Based on this analysis, we give recommendations regarding the best parameters to choose.

It is assumed that the nodes are able to calculate their position using GPS or other localization protocols [3]. Based on this information, they can determine the grid cell they belong to. Each grid cell chooses a representative that will be used to communicate with the representatives of the neighboring cells (above, below, left and right). In Figure 1 the cell representatives are marked in red.

Suppose that the communication range of all the nodes is r , then the grid size is chosen to be $d=r/\sqrt{5}$ so that the representatives of any two neighboring cells can communicate directly. The cell representatives use our algorithm to assign their channels. Any other node in a grid cell uses one of its representative's channels to communicate with it using a communication range $d\sqrt{2}$. The network is considered to be a dense network such that there is at least one node in each cell, therefore each cell has a representative.

All the nodes have C channels, Q radios and primary users could reclaim up to $k-1$ channels, where $k < Q < C$, so in the channel assignment k common channels are needed between the representatives of any two neighboring cells. The channels assigned to a node are represented as a vector of

dimension Q . The representatives are indexed like in a matrix, in order from top to bottom and from left to right.

We assign channels to representatives. The construction of the first row of the grid/matrix of vectors is presented next. We assumed $k < Q < C$ for an improved efficiency of our algorithm. We assign the channels 1, 2, 3, ..., Q to the representative in the cell (1,1) denoted by $node_{11}$ (top left corner in Figure 1).

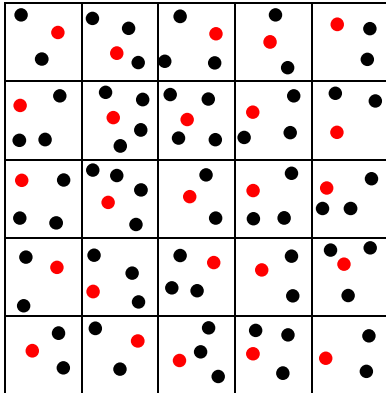


Figure 1: Monitored area divided into grids and cells' representatives (red)

Then, suppose we want k -connectivity, to support $k-1$ channels reclaimed by primary users, we assign the last k channels of $node_{11}$ to its right neighbor, $node_{12}$. So this neighbor will have the following channels assigned to its first k radios: $Q-k+1, Q-k+2, \dots, Q$. Since $k < Q$, there are $Q-k$ radios of $node_{12}$ that were not assigned. But $Q < C$ so there are additional channels available for this node. We continue assigning channels to the next radios in order, starting from $Q+1, Q+2, \dots$ and continuing until we exhaust all the

channels or assign all the radios using the remaining channels, whichever comes first.

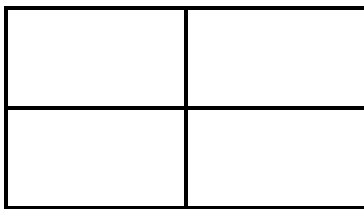
If we exhaust all the channels and there are radios that are unassigned, after assigning the last channel C we go back to channel 1 and we continue to assign channels in increasing order, starting with 1. The same method is applied for row 2, starting with $node_{21}$ that has the same assignment as $node_{12}$, so each element in row 2 will get the last k channels of the node above it and the next $Q-k$ channels in increasing order, where channel C is followed by channel 1.

The first table in Figure 2 shows an example, for $C=8, Q=5$, and $k=3$. We observe that starting from column 5 the elements start repeating, i.e. columns 1 and 5 are the same, columns 2 and 6 are the same etc. If we denote by A the 6×6 matrix of vectors represented in the first table of Figure 2, we have $A_{11}=A_{15}, A_{12}=A_{16}, A_{21}=A_{25}, A_{22}=A_{26}$ etc.

Matrix A is symmetric so the first row is the same as the first column. The rows start repeating from row 5. A cell from the second table in Figure 2 represents the 4×4 sub-matrix/block from the upper left corner of matrix A (marked by a thicker border). Such blocks will repeat horizontally and vertically in a large network where the representatives use this method to assign their channels.

Denote by $node_{1,j+1}$ the first node in the first row that has the same channels as $node_{11}$. The $j \times j$ grid of vectors is called *basic grid*. We computed the value of j for all the triples (C, Q, k) with C from 3 to 11, Q from 2 to $C-1$, and k from 1 to $Q-1$. This helped us to find the values for C, Q and k that give balanced distributions since the distribution of channels for basic grids could be used to find the distribution of channels for larger grids that have this basic structure repeated horizontally and vertically. The values for j could be stored or calculated using simple formulas. More details about this were given in [4].

1, 2, 3, 4, 5	3, 4, 5, 6, 7	5, 6, 7, 8, 1	7, 8, 1, 2, 3	1, 2, 3, 4, 5	3, 4, 5, 6, 7
3, 4, 5, 6, 7	5, 6, 7, 8, 1	7, 8, 1, 2, 3	1, 2, 3, 4, 5	3, 4, 5, 6, 7	5, 6, 7, 8, 1
5, 6, 7, 8, 1	7, 8, 1, 2, 3	1, 2, 3, 4, 5	3, 4, 5, 6, 7	5, 6, 7, 8, 1	7, 8, 1, 2, 3
7, 8, 1, 2, 3	1, 2, 3, 4, 5	3, 4, 5, 6, 7	5, 6, 7, 8, 1	7, 8, 1, 2, 3	1, 2, 3, 4, 5
1, 2, 3, 4, 5	3, 4, 5, 6, 7	5, 6, 7, 8, 1	7, 8, 1, 2, 3	1, 2, 3, 4, 5	3, 4, 5, 6, 7
3, 4, 5, 6, 7	5, 6, 7, 8, 1	7, 8, 1, 2, 3	1, 2, 3, 4, 5	3, 4, 5, 6, 7	5, 6, 7, 8, 1



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Figure 2: Channel assignment for $C=8, Q=5, k=3$ and repeating blocks in a large network

We consider a balanced distribution of channels to correspond to an assignment for which the distribution of channels assigned to all the nodes of a basic grid contains each channel from 1 to C an equal number of times (it is a uniform distribution). In [4] we showed how to construct a matrix that helps us to determine if the distribution of channels is balanced or not. A balanced distribution provides a fair utilization of channels. In this paper we analyze interference as another criterion that we could use when choosing the parameters C, Q and k for our algorithm.

2 The General Algorithm

Suppose that the vertex/node from the upper left corner of the grid, denoted by v_0 , has channel assignment $v_0 = (1, 2, 3, \dots, Q)$. Then the edges adjacent to it (right and bottom) have channel assignment $e_0 = (Q-k+1, Q-k+2, \dots, Q)$.

Based on GPS information, each representative assigns channels to its Q radios and then each horizontal edge gets the last k channels of the node to its left, and each vertical edge gets the last k channels of the node above it. The assignments will repeat since we can consider the network as basic grids put together side by side, horizontally and vertically. In what follows, J represents a vector of ones, with length given by its index.

We first consider channel assignment for the nodes of a basic grid of dimension $j \times j$. The channels assigned to the nodes from the first row/column with our channel assignment algorithm are given by $v_i = J_Q + ((v_0 + (i(Q-k)-1)J_Q) \bmod C)$, where $J_Q = (1,1,1,\dots,1)$ has length Q and i takes integer values from 0 to j-1, in increasing order. The channels assigned to the edges from the first row/column in our channel assignment algorithm are given by $e_i = J_k + ((e_0 + (i(Q-k)-1)J_k) \bmod C)$, where $J_k = (1,1,1,\dots,1)$, e_0 contains the last k elements of v_0 and i takes integer values starting from 0.

In what follows, the indices row, col are considered in the current basic grid while the indices x, y are considered in the whole network, which we suppose is a bigger grid. All indices start from 1 and they represent the coordinates of the representatives' cells in the matrix/grid of cells. Since the devices have GPS or other localization protocols [3], they are able to find x and y. Based on these, they can calculate indices row and col, and then assign their channels accordingly. Each node assigns its channels using the following algorithm.

Suppose that the left corner of each basic grid has assignment v_0 . In previous examples we considered $v_0 = (1,2,3,\dots,Q)$ but v_0 could be any assignment that contains k different channels. To find its assignment, an arbitrary node denoted by $node_{xy}$ calculates the indices row, col in the basic grid it belongs to: $row = 1 + ((x-1) \bmod j)$ and $col = 1 + ((y-1) \bmod j)$. Then, it finds the assignment for $node_{row, col}$ in the basic grid like this:

1. Calculates $w = 1 + ((row+col-2-1) \bmod j)$, where $row+col-2$ represents the Manhattan distance between the top leftmost node of its basic grid (or $node_{11}$) and $node_{row, col}$ in its basic grid, which is the basic grid corresponding to the node initially denoted by $node_{xy}$.

2. Finds the basic grid channel assignment for $node_{row, col}$ using the formula $v_i = J_Q + ((v_0 + (w(Q-k)-1)J_Q) \bmod C)$, where $J_Q = (1,1,1,\dots,1)$.

Suppose that each node $node_{xy}$ in the network assigns its Q channels using the previous algorithm. Then an edge incident to $node_{xy}$ that has this node on the left or above it, uses for communication the last k channels of this assignment. In other words, each horizontal edge uses the last k channels of the node to its left and each vertical edge uses the last k channels of the node above it.

This channel assignment method is robust to the presence of primary users that reclaim at most k-1 channels at once, no matter which channels these are. Other existing algorithms do not address robustness when multiple channels are reclaimed.

3 Analysis of Interference

Next we analyze the interference for the representatives' communication by considering the ideal case when these representatives form perfect grids and $d=r/\sqrt{5}$. We could obtain perfect grids if we can control the placement of the devices in the network. Our algorithm and results could be also used for grid networks.

In Figure 3 we consider a device/node u (black) placed in the middle of a network. The devices that are in the interference range of the node u are marked in red. Let us consider that u communicates with v (bidirectional communication). We add the edge $e = (u, v)$ and as a result we add additional nodes, marked in blue, that are in the interference range of v.

In [7] the authors used the following formula to calculate the potential interference index for an edge e: $p(e) = |\{(u', v'): (u', v') \in E, u' \text{ or } v' \in D(e)\}|$. E is the set of all communication edges and D(e) represents the set of nodes in the interference range of link $e = (u, v)$, i.e. $D(e) = D(u) \cup D(v)$.

We calculated the potential interference index of the edge e in the center for several values of C, Q and k, first by considering an edge as one link with k channels assigned to it ($p(e)$), and second, by considering that an edge consists of k links ($p'(e)$), each link having one of the k channels (common to the end nodes) assigned to it. We analyze interference just for horizontal edges because we assume a scheduling where in each time unit nodes transmit using either horizontal edges or vertical edges. Since matrix A is symmetric, the results for vertical edges are the same.

We assume that the assignment for the edge e is $e_0 = (a_1, a_2, a_3, \dots, a_k)$. In Figure 3 we marked with labels the horizontal edges that are in the interference range of e . All the edges marked with label 0 have the same assignment as edge e . Edges marked with the same label in Figure 3 have the same channels assigned to them. Next we use a vector of ones, denoted by J , whose index represents its length, i.e. J_k has length k .

In general, the edges marked with label i in Figure 3 have the assignment $e_i = J_k + ((e_0 + (i(Q-k)-1)J_k) \bmod C)$, where $J_k = (1, 1, 1, \dots, 1)$ and i can take integer values from -4 to 4 .

Let us assume that $C=10$, $Q=6$, $k=2$ and edge e from Figure 3 has the following channel assignment: $e_0 = (5, 6)$. Then we could use the previous formula to find the assignments for all the edges that are in the interference range of edge e : $e_1 = J_2 + ((e_0 + (1(Q-k)-1)J_2) \bmod C) = J_2 + (((5, 6) + 3 \cdot J_2) \bmod 10) = J_2 + ((8, 9) \bmod 10) = (9, 10)$, $e_2 = J_2 + ((e_0 + (2(Q-k)-1)J_2) \bmod C) = J_2 + (((5, 6) + 7 \cdot J_2) \bmod 10) = (3, 4)$ etc.

In Figure 3, the edges marked with 0 would have the assignment $e_0 = (5, 6)$, the edges marked with 1 the assignment $e_1 = (9, 10)$, the edges marked with 2 the assignment $e_2 = (3, 4)$ etc. In general, the edges marked with label d have the same interference with edge e as the edges marked with $-d$.

In the first formula for calculating interference ($p(e)$), if an edge that contains a node marked in red or blue has at least one channel in common with the edge e , we add 1 to the potential interference index of edge e . In the second formula ($p'(e)$), if an edge that contains a node marked in red or blue has t channels (one channel per link) in common with the edge e , we add t (instead of 1) to the potential interference index of edge e .

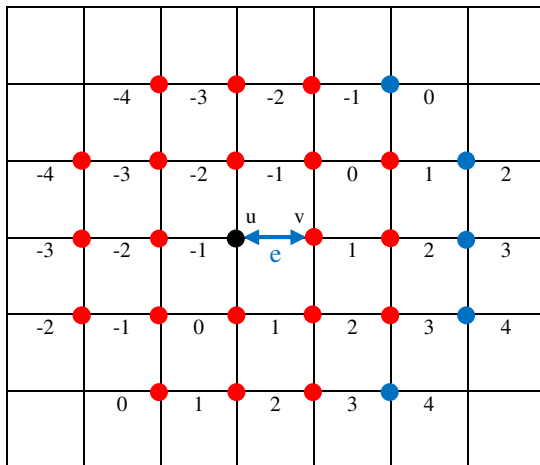


Figure 3: Horizontal edges that interfere with edge e

We calculated $p(e)$ and $p'(e)$ using a computer program that generates the channels for a grid and then calculates the interference. The results for $k=2$ and $k=3$ are presented in Figure 4b. We also derived some theoretical formulas that allow us to connect the values for $p(e)$ and $p'(e)$ to the values for the parameters C , Q and k , or more precisely, to the values n_i from Figure 4a.

We consider that an edge labeled with d ($d>0$) or $-d$ is at distance d or interferes with edge e at level d , where d can take any value from 1 to 4. Theoretical results related to the analysis of interference for any values of C , Q and k are given next.

If we denote by e_0 the channel assignment for edge e from Figure 3, then the edges that interfere at level d with edge e have the following channel assignment: $e_d = J_k + ((e_0 + (d(Q-k)-1)J_k) \bmod C)$. We observe that in Figure 3 we have four 0's, eight 1's, eight 2's, six 3's and four 4's as labels of edges that interfere with e .

If we denote by $n_i = (n_{i1}, n_{i2}, n_{i3}, n_{i4})$ the interference at levels 1, 2, 3, 4 for e , we can calculate the interference index $p'(e)$ like this: $p'(e) = 4k + (8, 8, 6, 4) n_i = 4k + (8, 8, 6, 4) (n_{i1}, n_{i2}, n_{i3}, n_{i4}) = 4k + 8n_{i1} + 8n_{i2} + 6n_{i3} + 4n_{i4}$.

Next we show how $n_i = (n_{i1}, n_{i2}, n_{i3}, n_{i4})$ can be calculated. The number n_{id} , d from 1 to 4, represents the number of common channels that exist between any two edges at distance d . For example, we choose the edges with assignments e_0 and $e_d = J_k + ((e_0 + (d(Q-k)-1)J_k) \bmod C)$. We could choose any two edges at distance d .

We consider a function S that converts the vectors e_0 and e_d to sets and we denote by \cap the intersection of two sets. This way we obtain $n_{id} = |S(e_0) \cap S(e_d)|$ where $| \cdot |$ denotes cardinality. Using these notations, we have $p'(e) = 4k + 8n_{i1} + 8n_{i2} + 6n_{i3} + 4n_{i4} = 4k + 8 \cdot |S(e_0) \cap S(e_1)| + 8 \cdot |S(e_0) \cap S(e_2)| + 6 \cdot |S(e_0) \cap S(e_3)| + 4 \cdot |S(e_0) \cap S(e_4)|$.

When calculating $p(e)$ we consider that two edges interfere if they have at least one common channel but we do not take into account how many channels they have in common. So $p(e) = 4 + 8b_{i1} + 8b_{i2} + 6b_{i3} + 4b_{i4}$, where $b_{id} = 1$ if $|S(e_0) \cap S(e_d)| > 0$ and $b_{id} = 0$ if $|S(e_0) \cap S(e_d)| = 0$. Figure 4 gives some results for n_i , $p(e)$ and $p'(e)$, colored red for imbalanced distributions.

4 Discussion of Results

We extended the analysis of a channel assignment algorithm that is robust to primary users who could reclaim multiple channels at once, and that can provide a balanced distribution of channels for certain parameters. We did a theoretical analysis of interference for different values of C , Q and k by considering that cell representatives form perfect grids, i.e. they are placed on a grid at distance d and their range is $r=d\sqrt{5}$.

C\Q	3	4	5	6	7	8	9	10
4	1, 0, 1, 2							
5	1, 0, 0, 1	0, 1, 1, 0						
6	1, 0, 0, 0	0, 0, 2, 0	0, 2, 0, 2					
7	1, 0, 0, 0	0, 0, 1, 1	0, 1, 0, 0	0, 1, 0, 0				
8	1, 0, 0, 0	0, 0, 0, 2	0, 0, 1, 0	0, 2, 0, 2	0, 0, 1, 0			
9	1, 0, 0, 0	0, 0, 0, 1	0, 0, 2, 0	0, 1, 0, 0	0, 1, 0, 0	0, 0, 2, 0		
10	1, 0, 0, 0	0, 0, 0, 0	0, 0, 1, 0	0, 0, 0, 0	0, 2, 0, 2	0, 0, 0, 0	0, 0, 1, 0	
11	1, 0, 0, 0	0, 0, 0, 0	0, 0, 0, 1	0, 0, 1, 0	0, 1, 0, 0	0, 1, 0, 0	0, 0, 1, 0	0, 0, 0, 1

k=2

C\Q	4	5	6	7	8	9	10
5	2, 1, 1, 2						
6	2, 1, 0, 1	1, 1, 3, 1					
7	2, 1, 0, 0	1, 0, 2, 2	0, 2, 1, 1				
8	2, 1, 0, 0	1, 0, 1, 3	0, 1, 2, 0	0, 3, 0, 3			
9	2, 1, 0, 0	1, 0, 0, 2	0, 0, 3, 0	0, 2, 0, 1	0, 2, 0, 1		
10	2, 1, 0, 0	1, 0, 0, 1	0, 0, 2, 1	0, 1, 1, 0	0, 3, 0, 3	0, 1, 1, 0	
11	2, 1, 0, 0	1, 0, 0, 0	0, 0, 1, 2	0, 0, 2, 0	0, 2, 0, 1	0, 2, 0, 1	0, 0, 2, 0

k=3

(a) Values of $n_i = (n_{i1}, n_{i2}, n_{i3}, n_{i4})$

C\Q	3	4	5	6	7	8	9	10
4	22 (30)							
5	16 (20)	18 (22)						
6	12 (16)	10 (20)	16 (32)					
7	12 (16)	14 (18)	12 (16)	12 (16)				
8	12 (16)	8 (16)	10 (14)	16 (32)	10 (14)			
9	12 (16)	8 (12)	10 (20)	12 (16)	12 (16)	10 (20)		
10	12 (16)	4 (8)	10 (14)	4 (8)	16 (32)	4 (8)	10 (14)	
11	12 (16)	4 (8)	8 (12)	10 (14)	12 (16)	12 (16)	10 (14)	8 (12)

k=2

C\Q	4	5	6	7	8	9	10
5	30 (50)						
6	24 (40)	30 (50)					
7	20 (36)	22 (40)	22 (38)				
8	20 (36)	22 (38)	18 (32)	16 (48)			
9	20 (36)	16 (28)	10 (30)	16 (32)	16 (32)		
10	20 (36)	16 (24)	14 (28)	18 (26)	16 (48)	18 (26)	
11	20 (36)	12 (20)	14 (26)	10 (24)	16 (32)	16 (32)	10 (24)

k=3

(b) Values of $p(e)$ and $p'(e)$

Figure 4: Results related to interference for specific parameters C, Q and k

When making decisions about choosing the best values for C, Q and k, we should take into account these results related to interference. If we have flexibility in choosing the values for C, Q and k, we should avoid the cases which give imbalanced distributions and/or the ones that have a high probability of interference.

Due to the large number of cases obtained by varying the values of the three parameters C, Q and k, results from simulations cannot be generalized without theoretical analysis and proofs. Interference analysis helped us to derive and prove useful results, one of which is the following: for $Q \geq 2k$ we cannot have interference at level 1 for our channel assignment. We also gave some formulas that help us to calculate interference for any values of C, Q and k.

Tables like those from Figure 4 help in making decisions regarding the number of channels (C) and the number of radios (Q) to use for our algorithm, based on the number of channels that are expected to be reclaimed by primary users at once (k-1). Depending on our goals and on the flexibility in choosing the values for C, Q and k, we could take into account both criteria (balanced distribution and small interference) or just one of them, in order to maximize the performance of the algorithm.

For example, if both criteria were taken into account, in Figure 4b we would give priority to black values (balanced distribution) that have small numbers (small interference). We constructed such tables for higher values of k also.

5 Conclusions

The results we obtained help when making decisions about the best values to choose for the parameters of our algorithm. Optimizing some network parameters like the number of radios per device would help in designing efficient hardware, in order to improve the network performance when running algorithms.

This channel assignment method is robust to primary users that reclaim multiple channels at once and it could be applied to different kinds of networks, including those with limited resources like wireless sensor networks. In

comparison to other algorithms that require exchanges of messages and cascaded switching of channels for multiple users, each device assigns channels to its radios based on simple calculations and this provides energy efficiency.

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