2D Modeling and Analysis of Railway Track under Subjected Loads*

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ABSTRACT

In this paper, a model is developed for dynamic analysis of a train track with two degrees of freedom. The track length is assumed to be 100 m in order to reduce the bindery effects. A wheel defect is modeled as a corrugation on the rail surface for obtaining dynamic responses of the rail. Two vehicle masses are used containing a locomotive as an unsprung mass and a wheel as a sprung mass. Rail element is modeled as a beam with vertical displacements and end rotations. In this model, sleepers are assumed as lumped masses 60 cm apart from each other. All the components, including rails, sleepers, ballast layers and rail vehicles are connected by springs and damping elements. Analysis of the model is performed by numerical solution through solving differential equations representing the entire system. In all equations, relations between the elements are taken as finite elements. Finally, all responses, displacements, velocities and accelerations of all degrees of freedom are obtained.

KEYWORDS: 2D modeling, Differential equations, Finite element, Railway track, Displacement, Velocity, Acceleration.

INTRODUCTION

Nowadays, high-speed railway systems have been built in many countries. These are considered as a preferred alternative to other kinds of transportation systems, especially for medium distances. There is an increasing need to improve safety, reliability and efficiency of railway tracks. Mechanical characteristics of tracks and dynamic vertical behaviors of tracks and wheels have been investigated.

For reaching high speeds while having more safety, the interaction behavior of rail/ vehicle is more important. Therefore, many models have been developed so far (Clark et al., 1982; Ahlbeck et al., 1975). Methods for studying the dynamic interaction of line/ rail vehicles can be divided into two categories, one of which is in the frequency domain and the other is in the time domain (Cai and Raymond, 1992). These types of analysis lead to avoid simultaneous solution of coupled differential equations (Oscarsson, 2001).

In the frequency domain, Timoshenko was the first who used Euler continuous beam in high-frequency analysis (Timoshenko, 1926). Output of calculations presents time domain diagrams, displacements, velocities and accelerations of radius elements of train and track (Zhai and Sun, 1994). In this present paper, all calculations are performed in time domain with solving all of the models by deformation equations.

Dynamic Model of Track by Using Finite Element Analysis

The dynamic model of the rail track as a beam element is presented in Fig. 1. In this model, the beam

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has two degrees of freedom at its two ends according to Equation (1).

$$\{x_i\} = \{u_i \quad \theta_{ix}\}; \qquad (1)$$

where the relation between forces $\{F\}^e$ and displacements $\{x\}^e$ is defined in Equation (2).

$$\{F\}^e = [k]^e\{x\}^e; \qquad (2)$$

where $[k]^e$ is the matrix of stiffness (Shrikhande, 2008).

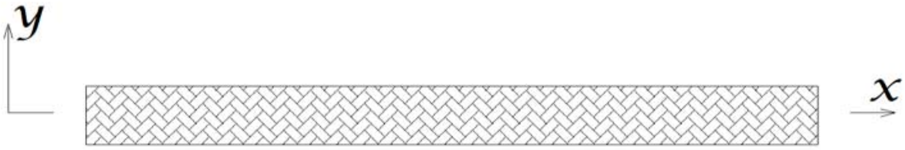

Figure (1): Beam element with two degrees of freedom

Mass matrix $[m]$ is defined in Equation (3) and is formed by the sum of masses of the elements.

$$[m]=\begin{bmatrix} m_1 & 0 & \dots & 0 \\ 0 & m_2 & \dots & 0 \\ .. & \dots & \dots & \dots \\ 0 & 0 & \dots & m_{Nj} \end{bmatrix}; \qquad (3)$$

where $m_1=\frac{\overline{m}l_1}{2}$, $m_{Nj}=\frac{\overline{m}l_j}{2}$ $\quad m_2=\frac{\overline{m}l_1}{2}+\frac{\overline{m}l_2}{2}$, ... ,

For damping matrix, Equation (4) is used (Zienkiewicz, 1997).

$$[c]^e=\alpha[m]^e+\beta[k]^e; \qquad (4)$$

where $[m]^e$ is the mass matrix and $[k]^e$ is the stiffness matrix of the beam elements. α and β are damping confections. By using Hamilton theory, the dynamic equations for solving the problem are as shown below (Kisilowski and Knothe, 1991).

$$[M]\{\ddot{x}_r\}+[C]\{\dot{x}_r\}+[K]\{x_r\}=\{Q\}; \qquad (5)$$

where:

$$[M] = \Sigma_e[m]^e; \quad [C] = \Sigma_e[c]^e; \quad [K] = \Sigma_e[k]^e; \quad [Q] = \Sigma_e[q]^e. \qquad (6)$$

In Equation (5), x_r, $\dot{x}_r$, $\ddot{x}_r$ are displacement, velocity and acceleration of the rail, respectively. In Equation (6), mass, damper, stiffness and force matrices are presented, respectively. In all equations, relations between elements are taken as finite elements (Shrikhande, 2008). Equation (7) presents the pattern of relations between elements in this equation. Carbody is shown as c, wheel as w, rail as r and sleepers are shown as s (i= 1,2,…, N_s). Also, in Equation (7), sub-matrices of $[c/w]$, $[w/r]$ and $[r/s]$ are relations between wheel and carbody, wheel and rail and rail and sleeper (Lei, 2001).

$$\begin{bmatrix} assemble & matrix & pattern \end{bmatrix} = \begin{bmatrix} [c] & \vdots & [c/w] & \vdots & [0] & \vdots & [0] \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ [c/w] & \vdots & [w] & \vdots & [w/r] & \vdots & [0] \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ [0] & \vdots & [w/r] & \vdots & [r] & \vdots & [r/s] \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ [0] & \vdots & [0] & \vdots & [r/s] & \vdots & [s] \end{bmatrix} \qquad (7)$$

Equation (5) is a differential equation that can be solved by classic methods, but because of the big size of the matrices, solving them by classic methods is not possible. Therefore, Wilson-θ numerical solution method is used in this paper. The rest of differential equations for rail vehicle, ballast and sleepers are assumed as below. The vehicle model is simplified as a locomotive mass and a wheel mass, where bogie masses are assumed on the wheel mass. In Fig. 2, the vehicle model on an elastic foundation is presented.

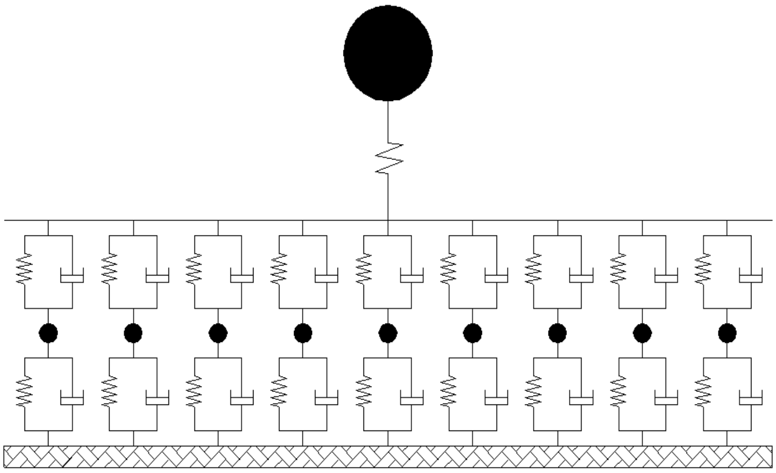

Figure (2): Track and vehicle model

Regarding Fig. 2, "*c*" and "*w*" are carbody and wheel set abbreviations, "k_s" and "c_s" are stiffness and damper of the suspension. "k_w" is Hertz spring's stiffness between wheel set and rail and is calculated by equation (8).

$$k_w = \frac{Q}{\delta}\,; \qquad (8)$$

where δ is calculated as shown in Equation (9).

$$\delta = [x_1(t) - x(x_1,t) - \bar{r}(x_1)]. \qquad (9)$$

In Equation (9), $x_1(t)$ is the displacement of wheel and $x(x_1,t)$ is the displacement of the rail. $\bar{r}(x_1)$ is the corrugation in the interaction area that is continuous function and is presented in Equation (14). According to Fig. 2, ky_1 and cy_1 are stiffness and damper of pad and ky_2 and cy_2 are stiffness and damper of ballast. The vehicle in the model has 2 degrees of freedom. Equations of all elements of the track are shown below.

Equation (10) is valid for vehicle.

$$-m\frac{d^2x_c}{dt^2} - k_s\left[x_c(t) - x_w(t)\right] - c_s\left[\frac{dx_c(t)}{dt} - \frac{dx_w(t)}{dt}\right] = 0\,. \qquad (10)$$

Equation (11) is valid for bogie and wheel.

$$-m\frac{d^2x_w}{dt^2} + k_s\left[x_c(t) - x_w(t)\right] + c_s\left[\frac{dx_c(t)}{dt} - \frac{dx_w(t)}{dt}\right] - Q(t) = 0\,; \qquad (11)$$

where $x_c(t)$ is the displacement of the vehicle and $x_w(t)$ is the displacement of the wheel and bogie.

Equation (12) is valid for the whole track.

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{Q\}; \qquad (12)$$

where, $[M]$, $[C]$ and $[K]$ are mass, damper and stiffness of track and vehicle system, and $\{x\},\{\dot{x}\},\{\ddot{x}\}$ are displacement, velocity and acceleration of the related joints, respectively. The relation of the sleeper displacement is presented in Equation (13).

The equation of motion of the i[th] sleeper is:

$$M_s\ddot{x}_{si} + (cy_{1i} + cy_{2i})\dot{x}_{si} + (ky_{1i} + ky_{2i})x_{si} - cy_{1i} \times \dot{x}_r - ky_{1i} \times x_r = 0 \quad (i=1,2,\ldots,N_s); \qquad (13)$$

where, x_{si}, $\dot{x}_{si}$ and $\ddot{x}_{si}$ are displacement, velocity and acceleration of the sleeper, respectively. For defining the corrugation of wheel flat modeling, Equation (9) is applied and Equation (14) is introduced (Lei, 2001).

$$\bar{r}(x_1) = 0.5D\,[\,1 - cos(2\pi y/L)]\; D = L^2/(16R_w); \qquad (14)$$

where "$\bar{r}(x_1)$" is the depth of wheel flat, "y" is the length direction of flat, "D" is the maximum depth of flatness, "L" is the length of flat and "R_w" is the radius of the wheel (Wu and Thompson, 2002).

Dynamic Response of Track under the Effect of High-speed Train

Responses of high speed vehicles with speeds of 200 km/h, 250 km/h and 300 km/h have been determined. The vehicle's model is presented in Fig. 3 and its specifications are shown in Table 1 (Lei, 2008). The track is a continuous track with UIC60 rail type and a mass of 60 kg/m.

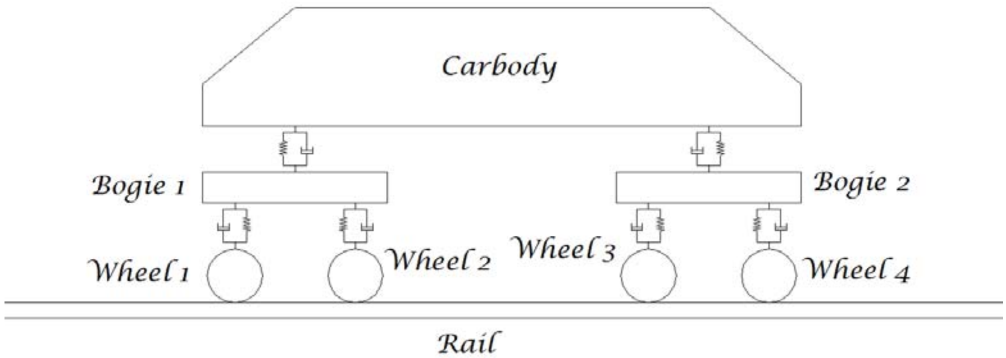

Figure (3): Locomotive axis of TGV

Other parameters are: area of rail (A = 0.717 × 10^{-2}m^2), inertia moment of rail (I=2.35×10^{-5}m^4), elastic modulus (E=2.07×10^8kN/m^2), rail density (ρ=7.83×10^3kg/m^3), stiffness coefficients of the sub-grade for pad and ballast layers (ky_1=20×10^4kN/m, ky_2=7.9×10^4kN/m) and damper coefficients of the sub-grade for pad and ballast layers (cy_1=70kNs/m, cy_2=50kNs/m). Hertz linear spring stiffness factor is considered to be k_w=1×10^3MN/m. Sleeper mass is m_p=250kg and thickness of ballast layer is H=35cm with lateral slope of 1:1.75 (Lei, 2008).

The parameter for calculating the numerical solution of differential equations of Wilson–θ method is θ=1.5 with time intervals of Δt=0.0001s. The coefficients α and β are the rail damping coefficients and are considered to be 0.0002 (α=β=0.0002) (Lei, 2008).

Time of passing is 2s (t_{total}=2s), which is enough to pass a bogie. The results of analysis are presented in Fig. 4 to Fig. 6 with various velocities of 200 km/h, 250 km/h and 300 km/h, respectively.

Table 1. Vehicle specifications

Parameter	***Value***
Axial load	170 kN
Wheel axis	3.0 m
Bogie mass	3.26 t
Stiffness of unsprung mass	1.31×10^3 kN/m
Stiffness of sprung mass	32.8×10^3 kN/m
Wheel radius	0.458 m
Vehicle mass	53.5 t
Wheel set mass	2.0 t
Damper of unsprung mass	30 kNs/m
Damper of sprung mass	90 kNs/m

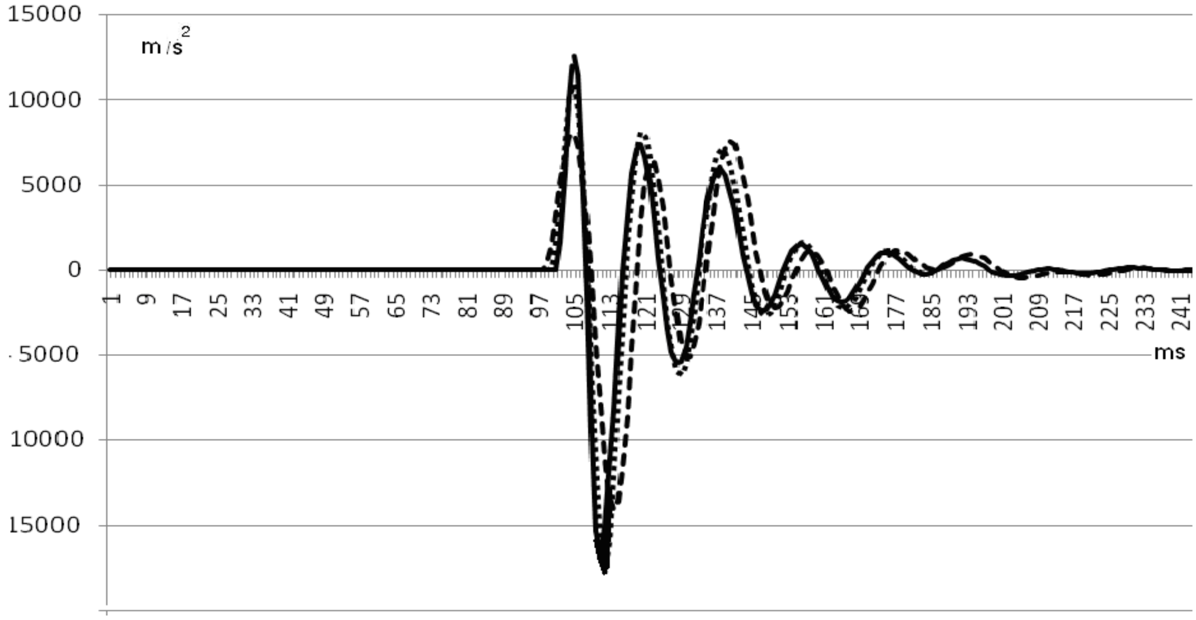

Figure (4): Acceleration of rail in wheel flat point for different velocities

300km/h ________

250km/h _ _ _ _ _

200km/h ………………

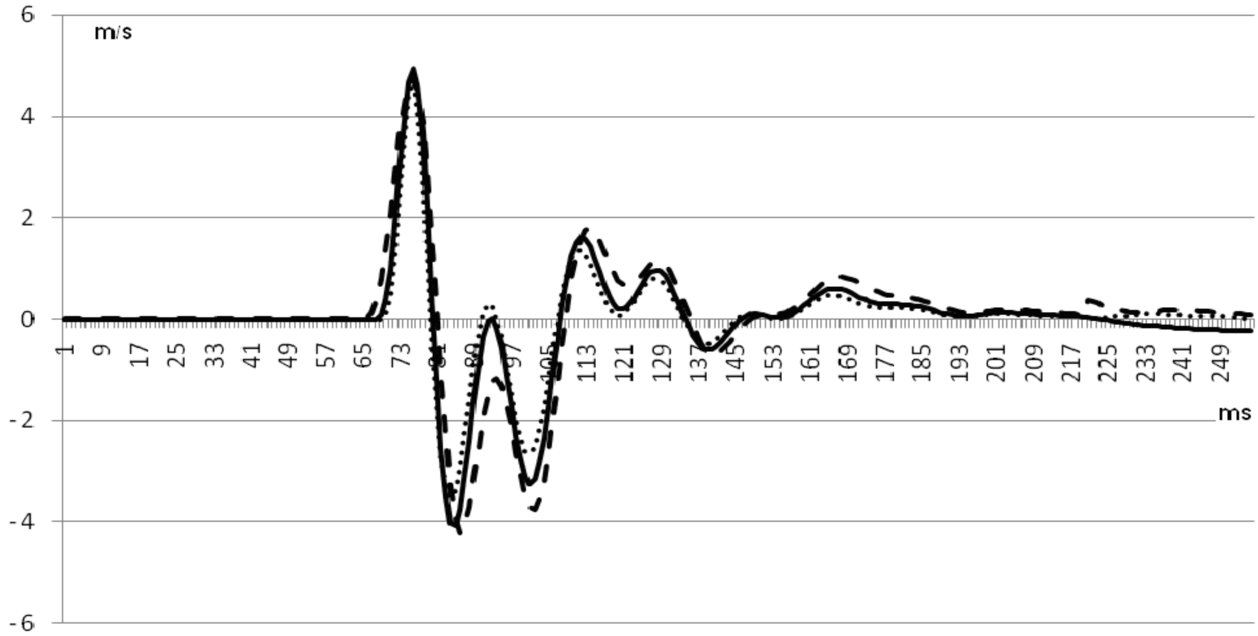

Figure (5): Velocity of rail in wheel flat point for different velocities

300km/h ________

250km/h _ _ _ _ _

200km/h ………………

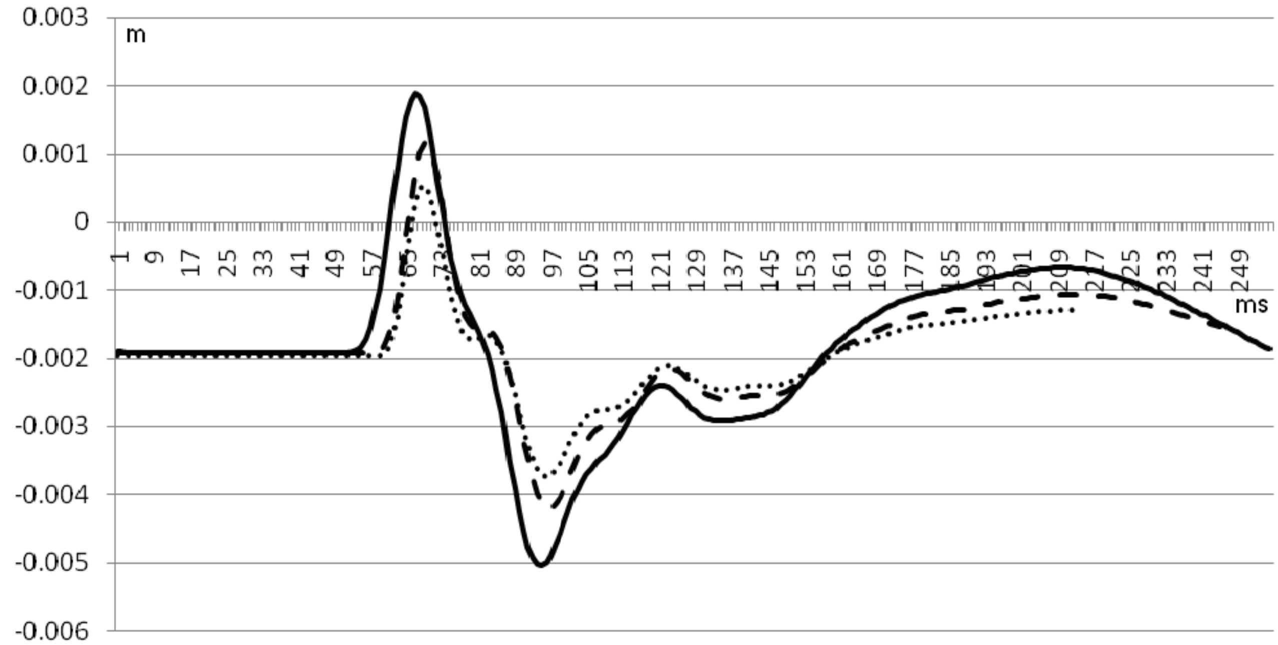

Figure (6): Displacement of rail in wheel flat point for different velocities

300km/h ―――――

250km/h _ _ _ _ _

200km/h ....

RESULTS AND DISCUSSION

According to this investigation, a dynamic model of track and vehicle is presented. Acceleration, velocity and displacement of the rail are presented for wheel flat with a depth of 100 mm and a length of 1mm for a 100 meters long track. Displacement, velocity and acceleration are calculated in the middle of the track. The results obtained are as follows.

A- According to Figs. (4-6), maximum displacement, velocity and acceleration in the track occur in the middle of the track when the wheel passes over flatness.

B- Maximum acceleration occurs at the velocity of 300km/h and is 16800 m/s^2. Decreasing the velocity to 250 km/h and 200 km/h causes an acceleration reduction of 5.98% and 17.29%, respectively.

C- Regarding the displacement of the middle of the track, it can be observed that loading and unloading in interaction areas occur at high speeds, whereby the dynamic forces of the contact area increase.

D- Regarding vibration velocity (Fig. 5), by crossing the wheel on the flat, a peak in the graph is created. The value of vibration velocity at the contact point of the wheel/rail at the speed of 300 km/h is calculated and found to be 4.9 m/s. For the remaining speeds, there are no significant changes.

E- According to Fig. 6, displacement at the velocity of 300 km/h is 1.9 mm. Decreasing the velocity to 250km/h and 200 km/h causes a displacement reduction of 39.47% and 73.68%, respectively. But, reconnecting the wheel and track causes an impact which increases the vertical displacement of the track more than 3 times of the previous value.

CONCLUSIONS

The main purpose of this study is to investigate the effects of wheel flatness behavior on railway tracks. The conclusions drawn from the study are listed below:

- Wheel flatness causes disruptions in the rail system, leading to maximum displacement, velocity and acceleration in tracks.
- Maximum displacement occurs when the flat part of the wheel is in contact with the track, because it causes an impact force on the track.
- High speeds of the train are more effective than low speeds. So, more deformations occur at higher speeds than at lower speeds.
- Increasing the dynamic forces of the wheel depends on the rapid progression of the vehicle and on the size of the wheel flatness.
- Because of the wheel flatness, understanding the dynamic behavior of the railway line is necessary.
- Wheel flatness causes not only dynamic displacements to the wheel and track, but also dynamic displacements and damages to the adjacent wheel.

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